- If $x_1(t) \xleftrightarrow{\text{LT}} X_1(s)$ with ROC R_1 and $x_2(t) \xleftrightarrow{\text{LT}} X_2(s)$ with ROC R_2, then

  $a_1x_1(t) + a_2x_2(t) \xleftrightarrow{\text{LT}} a_1X_1(s) + a_2X_2(s)$ with ROC R containing $R_1 \cap R_2$,

  where a_1 and a_2 are arbitrary complex constants.
- This is known as the linearity property of the Laplace transform.
- The ROC always contains the intersection but could be larger (in the case that pole-zero cancellation occurs.(

- If $x(t) \overset{\text{LT}}{\longleftrightarrow} X(s)$ with ROC R, then

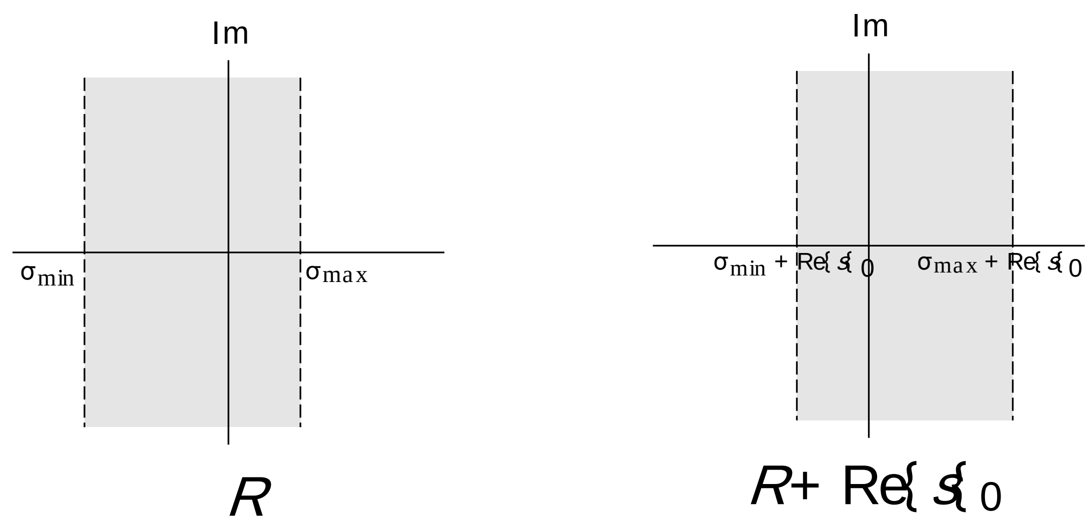
$$x(t-t_0) \overset{\text{LT}}{\longleftrightarrow} e^{-st_0}X(s) \quad \text{with ROC } R,$$

  where t_0 is an arbitrary real constant.
- This is known as the time-domain shifting property of the Laplace transform.

- If $x(t) \xleftrightarrow{\text{LT}} X(s)$ with ROC R, then

$$e^{s_0 t}x(t) \xleftrightarrow{\text{LT}} X(s-s_0) \quad \text{with ROC } R+\operatorname{Re}\{s_0\}$$

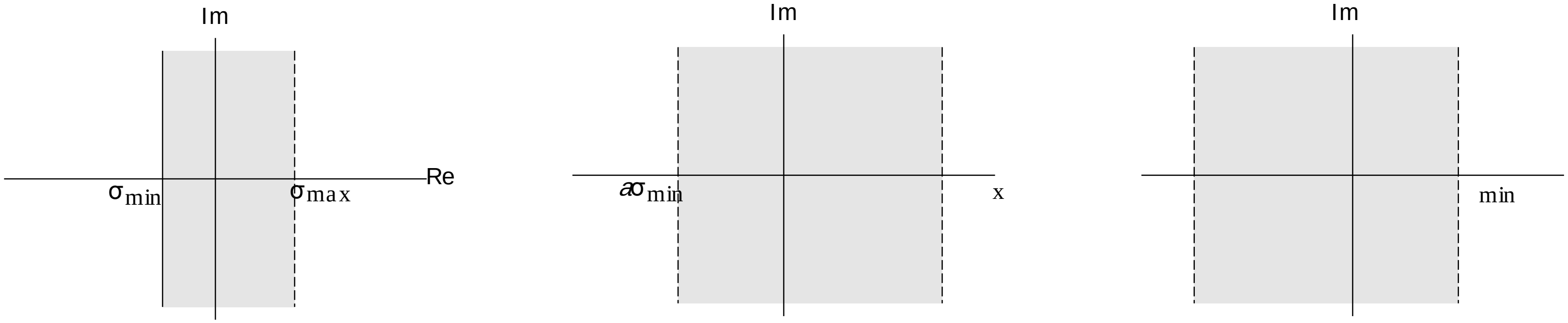
  where s_0 is an arbitrary complex constant.
- This is known as the Laplace-domain shifting property of the Laplace transform.
- As illustrated below, the ROC R is *shifted* right by $\operatorname{Re}\{s_0\}$.



- If $x(t) \overset{\text{LT}}{\longleftrightarrow} X(s)$ with ROC R, then

$$x(at) \overset{\text{LT}}{\longleftrightarrow} \frac{1}{|a|} X\left(\frac{s}{a}\right) \quad \text{with ROC } R_1 = aR,$$

  where a is a nonzero real constant.
- This is known as the (time-domain/Laplace-domain) scaling property of the Laplace transform.
- As illustrated below, the ROC R is *scaled* and *possibly flipped* left to right.



R $aR,\ a > 0$ $aR,\ a < 0$

- If $x(t) \overset{\text{LT}}{\longleftrightarrow} X(s)$ with ROC R, then

$$x^*(t) \overset{\text{LT}}{\longleftrightarrow} X^*(s^*) \quad \text{with ROC } R.$$

- This is known as the conjugation property of the Laplace transform.

- If $x_1(t) \overset{\text{LT}}{\longleftrightarrow} X_1(s)$ with ROC R_1 and $x_2(t) \overset{\text{LT}}{\longleftrightarrow} X_2(s)$ with ROC R_2, then

$$x_1 * x_2(t) \overset{\text{LT}}{\longleftrightarrow} X_1(s)X_2(s) \quad \text{with ROC containing } R_1 \cap R_2.$$

- This is known as the time-domain convolution property of the Laplace transform.
- The ROC always contains the intersection but can be larger than the intersection (if pole-zero cancellation occurs.(
- Convolution in the time domain becomes *multiplication* in the Laplace domain.
- Consequently, it is often much easier to work with LTI systems in the Laplace domain, rather than the time domain.

- If $x(t) \overset{\text{LT}}{\longleftrightarrow} X(s)$ with ROC R, then

$$\frac{dx(t)}{dt} \overset{\text{LT}}{\longleftrightarrow} sX(s) \text{ with ROC containing } R.$$

- This is known as the time-domain differentiation property of the Laplace transform.
- The ROC always contains R but can be larger than R (if pole-zero cancellation occurs).
- Differentiation in the time domain becomes *multiplication by s* in the Laplace domain.
- Consequently, it can often be much easier to work with differential equations in the Laplace domain, rather than the time domain.

- If $x(t) \overset{\text{LT}}{\longleftrightarrow} X(s)$ with ROC R, then

$$-tx(t) \overset{\text{LT}}{\longleftrightarrow} \frac{dX(s)}{ds} \text{ with ROC } R.$$

- This is known as the Laplace-domain differentiation property of the Laplace transform.

- If $x(t) \stackrel{\text{LT}}{\longleftrightarrow} X(s)$ with ROC R, then

$$\int_{-\infty}^{t} x(\tau)\,d\tau \stackrel{\text{LT}}{\longleftrightarrow} \frac{1}{s}X(s) \text{ with ROC containing } R \cap \{\text{Re}\{s\} > 0\}.$$

- This is known as the time-domain integration property of the Laplace transform.
- The ROC always contains at least $R \cap \{\text{Re}\{s\} > 0\}$ but can be larger (if pole-zero cancellation occurs.(
- Integration in the time domain becomes *division by s* in the Laplace domain.
- Consequently, it is often much easier to work with integral equations in the Laplace domain, rather than the time domain.

- For a function x with Laplace transform X, if x is *causal* and contains *no impulses or higher order singularities at the origin*, then

$$x(0^+) = \lim_{s \to \infty} sX(s),$$

where $x(0^+)$ denotes the limit of $x(t)$ as t approaches zero from positive values of t.
- This result is known as the initial value theorem.

- For a function x with Laplace transform X, if x is *causal* and $x(t)$ has a *finite limit* as $t \to \infty$, then

$$\lim_{t \to \infty} x(t) = \lim_{s \to 0} sX(s).$$

- This result is known as the final value theorem.
- Sometimes the initial and final value theorems are useful for checking for errors in Laplace transform calculations. For example, if we had made a mistake in computing $X(s)$, the values obtained from the initial and final value theorems would most likely disagree with the values obtained directly from the original expression for $x(t)$.

THIS SLIDE IS INTENTIONALLY LEFT BLANK.

Section 6.4

Determination of Inverse Laplace Transform

- Recall that the inverse Laplace transform x of X is given by

$$x(t) = \frac{1}{2\pi j}\int_{\sigma-j\infty}^{\sigma+j\infty} X(s)e^{st}\,ds,$$

  where $\mathrm{Re}\{s\} = \sigma$ is in the ROC of X.
- Unfortunately, the above contour integration can often be *quite tedious* to compute.
- Consequently, we do not usually compute the inverse Laplace transform directly using the above equation.
- For rational functions, the inverse Laplace transform can be more easily computed using *partial fraction expansions*.
- Using a partial fraction expansion, we can express a rational function as a sum of lower-order rational functions whose inverse Laplace transforms can typically be found in tables.

Section 6.5

Laplace Transform and LTI Systems